What are the goals for the K-5 math program?

If I could take a child, and pour into them what I think is important in terms of being successful in math, I would start with the ability to solve problems. So for me that is the first and foremost goal of the program. By the end of fifth grade we want to develop students that when presented with a problem take the attitude that given enough time they can solve it. In order for that to happen, students must have had many opportunities to practice solving problems, they must have the skills to work with numbers fluently and they must also have a good working knowledge of math concepts and vocabulary.

In our program, practicing problem solving begins right away; even before they formally begin school we pose problems for the children to solve in the Partners in Literacy and Numeracy program. We often only think of word problems when we think of problems, and we do work on those because the ability to take a word problem and transfer it into math concepts (Fosnot and Dolk call this "mathematizing" a problem) is very important, but any situation that the student does not already know how to solve is a problem to that child. Even a simple arithmetic equation such as 34 – 17 can be a problem to a student in first or second grade.

In order to solve problems, students sometimes need to be persistent. Not all problems are solved quickly and easily. The problem-solving workshop is an environment that gives students problems and gives them time to try and figure them out. Over time, we are trying to foster the belief that if you work hard on a problem and use the appropriate tools and resources you can figure it out.

You also need to have skills. Probably the most important skill is the ability to work with numbers fluently. This skill is called computational fluency and is more than just the ability to get the exact right answer using a paper and pencil procedure for each operation. I remember a few years back doing a problem about running laps around a soccer field. The question was given the dimensions of the field figure out how many laps you would need to run in order to run more than a mile. I had done this problem in many classrooms and at the end of the second week a teacher solved it mentally by figuring out the perimeter and then by doubling and doubling again seeing that four times the perimeter was just a little bit too small so it had to somewhere in the fifth lap that you reached a mile. In seconds, he had answered the question using an estimation strategy that no one else had thought to use in the previous two weeks. We are so programmed to using a certain procedure that we often forget to think first. Our goal is train students to have the skill of calculating exact answers using paper and pencil procedures but more than that we want students who look at the problem and look at the numbers and make some intentional choices.

- Do I need the exact answer or is an estimate good enough for this situation?
- Should I do this calculation mentally, using paper and pencil or use a calculator?
- Do I know any relationships that would make it easier to calculate the answer? For instance, to solve 75 – 49 = ☐ a student may use the easier fact that 75 - 50 = 25 to figure out that 75 – 49 = 25 + 1 or 26

When students ask themselves questions like these before they do the calculation, I think they are displaying the qualities that truly define computational fluency.

Students also need to have a large reservoir of concepts and vocabulary that they can call upon to help them solve problems. More than that though they need to realize that if they forget a concept or term they have the ability to use reference tools such as dictionaries, almanacs, and textbooks to refresh their memory. Exactly what concepts and vocabulary a person needs is determined by the problems and work they are doing, but the State Department of Education has outlined what they think students should have mastery of at each grade level by developing the Connecticut Mastery Test. Another resource that spells out what is expected at each grade level is a document called the Frameworks Companion. Recently, the state released an even more detailed version of the Frameworks called Grade-Level Expectations (GLEs). The Connecticut Framework Standards and Grade-Level Expectations present the mathematics content and concepts that students need to know in order to learn mathematics.

These resources are available from the Connecticut State Department of Education Web Site. When the link opens you will need to click on the resource you are interested in.

At the national level, the National Council of Teachers of Mathematics (NCTM) released a document called the focal points that spells out what the primary emphasis should be at each grade level.

So if those are the ultimate goals, what are the grade level expectations along the way?

In terms of developing problem solving we pose problems that we feel are appropriate for the students at each grade and time of year.

For the early elementary grades we use word problems based on the Cognitively Guided Instruction problem structure to help our students learn about addition

and subtraction but we also use problems that develop other concepts such as time and money.
In the upper elementary grades, we use problems from the Math Forum as well as activities that are problematic for our students from a variety of sources including Math Lands, Investigations in Number, Data, and Space, and technology such as the National Library of Virtual Manipulatives.

To develop computational skills, we have a time line that we expect will match the development of almost all of our students. As with any curriculum, there will be exceptions and some children will develop faster or slower than we expect. We use activities that are flexible and expandable to account for these differing rates of development. For example, when solving problems, some students will use concrete objects to figure out the solution while others will not need the manipulatives so they will figure out the solution mentally or using a paper and pencil procedure. When practicing skills such as fluency with number combinations (basic facts), students can be doing the same activity but working at different levels. One student may be doing an activity that focuses on figuring out what are the combinations for numbers up to 6 while another student in the same class may be working on combinations for numbers greater than 10. We want all of our students to get to the same place, but realize that they may get there at different times and in different ways.

Pre-Kindergarten to Second Grade
We look at Pre-Kindergarten through the Second Grade as one segment of a child's development. What makes this time so difficult for teachers is that students' brains are developing at different rates just as their bodies will develop at different rates in adolescents. In my mind, this is one of the most critical times in a child's development because they are trying to become fluent with words and numbers. The problem is sometimes as adults we do not remember how hard it was to become fluent and so we lose patience or have unfair expectations and rush the process. Kids need lots of opportunities to learn to read and they also need lots of opportunities to learn about our number system. Therefore the major focus of this time period is for students to become fluent with our number system. Kathy Richardson has identified 9 critical understandings for children to acquire before they are able to work with numbers as adults do. The most important of these understandings is place value. If students do not understand how where a digit is placed in a number changes the value of that digit, they will not be able to use paper and pencil procedures with understanding. Our goal for the end of second grade or the fall of third grade is that students have acquired these critical understandings. The progression within this time period will vary based on the child, but the typical progressions is as follows:

Pre-Kindergarten and Kindergarten:

- Learning to count objects and knowing one more and less without having to count
- Solving addition and subtraction problems by acting them out or drawing models of the action

First Grade:

- Becoming automatic with number combinations with sums up to 10
- Understanding the concept of addition and subtraction
- Solving addition and subtraction problems and recording the action using symbols such as 8 + 7 = 15

Second Grade:

- Becoming automatic with number combinations with sums up to 20
- Solving addition and subtraction problems with 2-digit numbers. Students may or may not use traditional methods of recording their thinking at this grade level. One popular alternative to the traditional procedure is the open number line.
- Have a conceptual understanding of tens and ones (place value)
- Late in the year, begin to solve multiplication and division problems by acting them out or drawing models of the action and matching up the appropriate symbols such as 25 ÷ 5 = 5

By the end of second grade we expect students to be automatic with all the number combinations (basic facts) for addition and subtraction and to be able to solve problems with double-digit numbers. At this point we are not as concerned with efficient methods of calculating sums and differences as much as we are interested in knowing if the child understands what they are doing. Therefore, their methods may seem cumbersome to us as adults, but the transparency of the process to the child is paramount. A very good article about developing computational procedures with understanding is an article by Susan Jo Russell, "*Developing Computational Fluency with Whole Numbers*" that appeared in ***Teaching Children Mathematics*** (November, 2000).

Third to Fifth Grade

In this grade band, we are transitioning from conceptual based procedures (such as counting up by ones) to more efficient procedures. This is work that requires lots of repetition. We used to try and teach these procedures in units like we teach concepts, but that was not working. We have taken a new approach; we use mini-lessons or routines for 10 to 15 minutes each day that focus on developing efficiency. Each grade has a focus. In this way, students have long windows of opportunity to learn to become efficient with a procedure for each operation (addition, subtraction, multiplication, and division). The San Diego Public Schools have created a resource that gives examples of some routines.

When the link opens, click on the word "Routines" for a listing of types of routines sorted by grade level. In addition to the ones they list we also use word problems and estimation problems as mini-lessons or routines. These mini-lessons and routines involving numbers begin when the majority of students in a class show they have an understanding of place value. For most classes, this will happen some time in second grade (probably in the second half of the year). Prior to that, we use mini-lessons and routines that focus on skills such as reading the calendar, telling time, naming coins, or counting and number activities such as counting straws to figure out how many days we have been in school.

Third Grade:

- Become fluent with adding and subtracting multi-digit numbers through mini-lessons
- Conceptually learn that multiplication is a quicker way of doing repeated addition and division is a quicker way of doing repeated subtraction or grouping
- Begin to become automatic with multiplication and division facts
    - Start with factors of 0, 1, 2
    - Grow to 5 and 10
    - By end of year should be automatic with above as well as 3 and 4
- Conceptually begin work with fractions and decimals (concrete tools/models/symbols)
    - Concrete Tools are objects used to represent a concept such as Fraction Circles
    - A model is a diagram used to represent a concept such as ☐☐☐☒ to represent ¾
    - Symbols are the math notation we use to represent a concept such as ¾ + ½ = 1 ¼

Grade 4:

- Continue to practice multi-digit addition and subtraction to make sure students are computationally fluent with this action
- Continue work with becoming automatic with multiplication and division facts. By end of year students should be automatic with all facts
- Begin work on becoming fluent with multiplication and division procedures through mini-lessons
- Conceptually begin work with fractions and decimals (concrete tools/models/symbols)

Grade 5:

- Hopefully facts are automatic by this point, but students that are not yet automatic may need intervention support to help them become automatic

- Continue to practice addition and subtraction in mini-lessons to maintain fluency
- Continue work with becoming fluent with multiplication and division
- Begin work on becoming fluent with fractions and decimal procedures through mini-lessons

We have expectations for the number strand at each grade level because it is skill based and is usually developed in a somewhat linear basis.

The other strands are developed in a more spiral-based nature. This means they are introduced at the early grades but mastery is not expected because these strands are conceptual in nature and people do not ever master them but grow to develop deeper and deeper understandings over time. Even as adults, we have the capacity to learn more about these topics.

The specific topics that are introduced at each grade level are spelled out in the Frameworks and Grade-Level Expectations (GLEs), but the general topics that are included in these other strands are:

- Data, Statistics, and Probability
    - Gathering and representing data using tables, charts and graphs
    - Interpreting Data using statistics such as mean, median, mode and range
    - Probability
- Algebraic Concepts
    - Exploring the concept of equality using tools such as the pan balance
    - Patterning (both repeating and growing patterns)
    - Beginning to use symbols (variables) in equations. For example, to represent the idea that the number of wheels needed for any number of cars can be figured out using the formula: four times the number of cars is the number of wheels needed, students might write (4 * C) = W or C + C + C + C = W
- Geometry and Measurement
    - Learning the vocabulary that we use to name and describe objects in our world such as square, cube and symmetrical
    - Learning to sort objects based on their attributes
    - Learning to put together and take apart shapes and objects
    - Learning to represent three dimensional objects two dimensionally and to create scale drawings
    - Learning about how objects can be moved (Transformations: slides, flips, and turns)
    - Learning to describe objects based on their size, area, or weight
    - Learning to use measurement tools such as rulers, protractors, and scales